Effiziente Algorithmen

Summer term 2012

Prof. Dr. Georg Schnitger, Bert Besser

Arbeitsgruppe Theoretische Informatik, Institut für Informatik

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Assignment 1

Take home: 16.04.2012
Submit: 23.04.2012

Note: The maximum score for this assignment is 24. At most three of the four exercises can contribute to your score. Please make sure to clearly mark those exercises you wish to be relevant for your score.

Exercise 1.1. (8)

Conditional probabilities

A pharma corporation introduces a new test for a certain genetic defect. The *false-negative* rate is very small: If a subject suffers from the defect the test will fail with a probability of 0.001. The *false-positive* rate is small as well: A subject without the defect passes the test with a probability of 0.005. Assume that 2% of the population suffers from the defect.

What is the probability that the classification of a subject as having the defect is wrong?

Exercise 1.2. (8)

Expectation of indepentent variables

Show that $E(X \cdot Y) = E(X) \cdot E(Y)$ holds for independent random variables X, Y.

Exercise 1.3. (8)

At the casino

At the casino we play a game with success probability $p = \frac{1}{2}$. We may bet an arbitrary amount. If we win we gain twice the bet. Otherwise we lose the bet. Consider the following strategy:

```
i = 0
REPEAT UNTIL won
  bet 2^i
  i = i + 1
```

What is the expected number of rounds until the first win?

What is the expected win, i.e. the expected difference between the total invested money and the obtained money? What is the expected liquidity, i.e. the expected amount that must be bet until the first win?

Exercise 1.4. (8)

A number game

We play a game against an adversary A. Without our knowledge A chooses two different numbers from the set $\{0, 1, \ldots, MAXINT\}$. We select one of A's numbers uniformly at random. After inspection we may choose to discard the first number and select the remaining one instead. Let x be the selected number and y the other one. Our gain is $x - y$, which might be negative.

Consider the following strategy for a given threshold t. We discard the first selected number iff $x < t$. What is the expected gain given x, y and t?

State a strategy for an expected positive gain for *any* arbitrary x, y.