# Effiziente Algorithmen

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Prof. Dr. Georg Schnitger, Bert Besser

Arbeitsgruppe Theoretische Informatik, Institut für Informatik

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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# Assignment 4

Take home: 05/07/2012
Submit: 05/14/2012

*Note*: The maximal score for this assignment is 16. Additional points will be added to the total score of all assignments.

### Exercise 4.1. (8)

*Bit fixing on hypercubes*

We look at a particular packet $p$ while executing the bit fixing strategy. Let $P$ be the set of packets other than $p$ whose paths have at least one edge in common with $p$'s path.

Show that the total number of steps that $p$ has to wait in one of the queues on its path is at most $|P|$.

### Exercise 4.2. (8)

*Fingerprinting*

Given three matrices $A, B, C \in \mathbb{Z}^{n\times n}$ we want to test whether $AB \neq C$. Assume that the arithmetic operations + and $\cdot$ take constant time when applied to numbers from $\mathbb{Z}$.

State an algorithm with one-sided error that runs in $O(n^2)$ time and prove its correctness.

*Hint*: Use the fact that for any $x \in \mathbb{Z}^n$ at most half of the vectors $s \in S = \{0,1\}^n$ satisfy $xs = 0$, where $xs$ denotes the scalar product $\sum_{i=1}^{n} x_i s_i$.

### Exercise 4.3. (8)

*The probabilistic method*

Consider a board of $n \times n$ cells, where $n = 2k, k \geq 2$. Each of the numbers from $S = \{1, \cdots, \frac{n^2}{2}\}$ is written to two cells so that each cell contains exactly one number.

Show that $n$ cells $c_{i,j}$ can be chosen with one cell per row and one cell per column such that no pair of cells contains the same number.

*Hint*: What about random permutations?