

## Assignment 5

Take home: 05/14/2012

Submit: 05/21/2012

### Exercise 5.1. (8)

*Making random sources uniformly distributed*

Our task is to build a random source that outputs the bits 0 and 1 with  $\text{prob}(0) = \text{prob}(1) = \frac{1}{2}$ . We have access to another random source  $S$  that outputs  $a$  or  $b$  with independent probabilities  $\text{prob}(a)$  and  $\text{prob}(b) = 1 - \text{prob}(a)$  that are unknown to us.

State an algorithm that does the job and that does not consume more than an expected number of  $(\text{prob}(a) \cdot \text{prob}(b))^{-1}$  symbols of  $S$  between two output bits. Prove its correctness.

### Exercise 5.2. (8)

*Fingerprinting*

Two processors  $A, B$  with inputs  $a \in \{0, 1\}^n$  (for  $A$ ) and  $b \in \{0, 1\}^n$  (for  $B$ ) want to decide whether  $a = b$ .  $A$  does not know  $B$ 's input and vice versa.

$A$  can send a message  $m(a) \in \{0, 1\}^*$  which  $B$  can use to decide  $a = b$ . The communication and computation rules are called a *protocol*.

- Show that every deterministic protocol must satisfy  $|m(a)| \geq n$ .
- State a randomized protocol that uses only  $O(\log_2 n)$  Bits. The protocol should always accept if  $a = b$  and accept with probability at most  $\frac{1}{n}$  otherwise. Prove its correctness.

### Exercise 5.3. (8)

*Continuous uniform samples*

A source provides a stream of items  $x_1, x_2, \dots$ . At each step  $n$  we want to save a random sample  $S \subseteq \{(x_i, i) | 1 \leq i \leq n\}$  of size  $k$ , i.e.  $S$  should be a uniformly chosen sample from all  $\binom{n}{k}$  possible samples consisting of seen items. So at each step  $n \geq k$  we must decide whether to add the next item to  $S$  or not. If so we must also decide which of the current items to remove from  $S$ .

State an algorithm for the problem. Prove its correctness.