

Effiziente Algorithmen

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Assignment 6

Take home: 05/21/2012

Submit: 05/29/2012

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 6.1. (8)

Closest pair

Show how to implement step (2b) of Algorithm 2.21 in expected time $O(|P_i|)$.

Hint: Use universal hashing and in particular apply Theorem 2.20.

Exercise 6.2. (8)

Streaming data

We receive a stream of $n - 1$ pairwise different numbers from the set $\{1, \dots, n\}$.

Show how to determine the missing number with an algorithm that reads the stream once and uses a memory of only $O(\log_2 n)$ bits.

Exercise 6.3. (8)

Clique sizes in random graphs

We consider the family of random graphs $G(n, p)$ with n nodes. Each possible edge is independently inserted into $G(n, p)$ with probability p . Let X_k be the number of cliques of size k in $G(n, p)$.

- Prove that $E(X_k) = \binom{n}{k} \cdot p^{\binom{k}{2}}$.

Now let $p = \frac{1}{2}$, i.e. we consider the family $G(n, \frac{1}{2})$.

- For $n \rightarrow \infty$ show that $E(X_{\log_2 n}) \geq 1$.
- For $n \rightarrow \infty$ show that $E(X_{c \cdot \log_2 n}) \rightarrow 0$ for a constant $c > 1$ to be found by you.

Conclusion: Random graphs will contain only small cliques w.h.p.

Hint: $\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{n \cdot e}{k}\right)^k$.