Effiziente Algorithmen

Summer term 2012

Prof. Dr. Georg Schnitger, Bert Besser

Arbeitsgruppe Theoretische Informatik, Institut für Informatik

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Assignment 8

Take home: 06/04/2012
Submit: 06/11/2012

Note: It is understood that all of your statements have to be proven correct.

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 8.1. (8)

Stationary distribution

Consider the Markov chain on the state set $\{0, 1, \ldots, n\}$. The transition probabilities are $P_{n\to 0} = P_{n\to n} = P_{i\to 0} = P_{i\to i+1} = \frac{1}{2}$ where $i < n$. State the chain's stationary distribution.

Exercise 8.2. (4+4)

Odyssey

Consider the Markov chain on the state set $\{0, 1, \ldots, n\}$ where $n = 2k$. The transition probabilities for state i $(0 < i < n)$ are $\frac{1}{2}$ to state $i + 1$ and $\frac{1}{2}$ to state $i - 1$. The chain has reflecting ends, i.e. $p_{0,1} = p_{n,n-1} = 1$.

- State the stationary distribution.
- What is the expected number of steps to reach state n starting from state 0?

  Hint: Can you derive a recurrence equation for the expected number e_i of steps to reach state n from state i?

Exercise 8.3. (8)

Randomized 2-SAT

Assume that the 2-SAT formula f on n variables has a solution. Consider the following algorithm.

```
compute a random variable assignment
while ∃ unsatisfied clause
 select an unsatisfied clause c uniformly at random
 select one of c's variables x uniformly at random
 invert x
```

Analyze the expected number of iterations of the loop until a satisfying variable assignment is found.

Hint: Design a Markov chain to model the behaviour of the algorithm.

Conclusion: This randomized algorithm is not as fast as tailor-made (linear time) algorithms but it is very simple and generalizes to the case of 3-SAT formulae, as we will see later on.