



# Parallel and Distributed Algorithms

Winter 2009/2010

8

Issue: 7.12.2009

Due: 14.12.2009

## Information

Solutions in english or german are fine.

### 8.1. Problem (8)

*Mandelbrot Set*

For a complex number  $c$  we define the following iteration

$$z_0(c) = 0, z_{k+1}(c) = z_k^2(c) + c.$$

The Mandelbrot Set  $\mathcal{M}$  consists of all numbers  $c$  such that the sequence  $z_k(c)$  is bounded, i.e., the set  $\{|z_k(c)| \mid k \in \mathbb{N}\}$  is contained in some ball with center zero. One can show  $|c| \leq 2$  for all  $c \in \mathcal{M}$ .

Assume  $|z_k(c)| = 2 + \epsilon$  for some  $\epsilon > 0$ . **Show** that  $|z_{k+1}(c)| \geq (1 + \epsilon)|z_k(c)|$ . (As a consequence  $c$  does not belong to  $\mathcal{M}$  whenever  $|z_k(c)| > 2$  for some  $k$ .)

### 8.2. Problem (8)

*Asynchronous Round Robin*

We consider the asynchronous round robin method for work distribution among  $p$  processes. Each process  $p_i$  has a local target variable pointing to exactly one process. Initially target variables are set arbitrarily.

Whenever a process requests work, it accesses its local target variable, requests work from the specified process and increments its target variable by one modulo  $p$  (whether the request has been granted or not).

If a process with work  $\leq \frac{W}{p}$  receives a request it will not share its work.

**Show** that  $\Theta(p)$  rounds are required until at least  $p$  processes have received work, provided the target values are initialized in a worst-case fashion.

### 8.3. Problem (8)

*Work Stealing for Backtracking*

Consider Remark 6.1 on page 115 and **show** that any parallel algorithm that traverses a tree  $T$  by a depth-first traversal has to spend at least  $\max\{\frac{N}{p}, h\}$  steps, where  $N$  is the total number of nodes of  $T$ ,  $h$  is the depth of  $T$  and  $p$  is the number of processes used.